

A Real Options Approach to Contractual Agreements and Value Flexibility

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Abstract

Contracts are usually analyzed in the light of the reduction of transaction costs that they may ensure. But this disregards the advantages of strategic flexibility in business relations. In this paper we consider a model of provider-client relation and see how flexibility in the contract (seen as a combination of a put and a call option) ensures a higher payoff to the involved parties.

Keywords: Contracts, Transaction Costs, Real Options

1. Introduction

A firm can be seen, abstractly, as a portfolio of agreements with outside partners, yielding costs and benefits. The firm lowers its exposition to uncertainty by means of rigid contracts, but their positive effects are overshadowed by the corresponding loss of strategic flexibility: expected benefits from future business opportunities can be lost due to the binding obligations that force their rejection.

The advantages of contracts, particularly those intended to protect investment in specific assets has been predicated in terms of reduction of transaction costs (Williamson, 1985), (Rese and Roemer, 2004). The protection is obtained through low-yield, transaction specific investments, covering the risks derived from three possible sources: malicious behavior of other agents, contingencies of the markets or changes in technology. Most of this literature treats only static models, focusing on behavioral risks.

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But this approach disregards the maneuvering possibilities of benefiting the strategic flexibility provided by the dynamic of potential markets opportunities.

This paper intends to suggest ways to enhance contracts by means of Real Options. We derived decision making-models in which a balanced is reached between protection (with is concomitant loss of flexibility) and openness to business opportunities. Our approach complements the literature on strategic trade-offs between flexibility and contracts (Argyres and Liebeskind, 1999); (Argyres and Liebeskind, 2000); (Barney and Lee, 2000); (Rese and Roemer, 2004). A brief discussion of these references can help to put into context our own take the problem. (Argyres and Liebeskind, 1999);(Argyres and Liebeskind, 2000) explore the consequences of contracts over corporate governance, claiming that real options are the right tools for the designs of corporate structures. (Rese and Roemer, 2004), in turn, develops a binomial options model that trade-off agreement and strategic flexibility. Following this lead we will consider a binomial model of valuation of exchange options on provision contracts. Considering complete information firm-clients games, we will see that an adequate balance between profits and punishments allow support, in Nash equilibria, both the enforcement of contract and the adequate means to breach them when better alternative becomes available.

The plan of the paper is as follow. Section 2 compares the literature on transaction costs with the recommendations on real option analysis, in order to see which aspects should be taken from each of these two approaches. Section 3 develops an example in which a stepwise analysis shows that and adequate degree of flexibility can be good for both parties and contracts. Section 4 draws the conclusions of the exercise and concludes.

2. Transactions Cost Analysis vs. Real Options

Transaction Cost Analysis (TCA) seeks to design efficient mechanisms, minimizing transactions costs (Williamson, 1983), (Williamson, 1985). The ensuing contracts are intended to protect economics relations among the agents. Some of their associated costs are due to the transactions leading to agreements. The main sources of transactions costs considered in the literature are:

- *Bounded rationality*: agents are assumed to have limited capacity of acquiring and processing information, restricting their self-interested decision-making abilities.

- *Generalized uncertainty*: while the intentional behavior of other agents is its main source, the business context in which the firm acts (the economy, the production sector to which it belongs, the technology, etc.) adds more uncertainty to decision-making.
- *Specificity of transactions*: non-specific liquid assets provide efficient mechanism supporting exit or sale options. Exiting is more costly for specialized and less liquid assets that demand extra provisos for the protection of investments.

Of these, the existence of investments in specific assets and the pervasiveness of uncertainty are, perhaps, the more important factors. The former involves assets satisfying only specific exchange relations, having low recovery value outside those relations. They are risky in the sense that their excess value can be appropriated by the business counterparts of the firm (Klein, Crawford and Alchian, 1978). Transactional uncertainty, on the other hand, arises from unforeseen contingencies. For instance, in the main application of this paper, namely provider-client relations, it amounts to the difficulty of predicting the volume that will be actually demanded to the supplier, due to volatility of the market in which the client operates. The ensuing renegotiations induce to extra costs to be accounted for in the contracts.

While exchanges relations could arise without any previous agreement, TCA prescribes vertical integration as a way of minimizing transaction costs, provide market by agreed-on buyer-supplier actions in coordinated fashion (Heide and Stump, 1995). The advantages of vertical integration are evident in stable business contexts, where transactions dynamics and the possibility of new opportunities can be disregarded (Rese and Roemer, 2004) If the latter is not the case it becomes necessary to allow degree of strategic flexibility.

A possible way of achieving strategic flexibility is by means of options and concomitant incentives to respect (or break) contracts. The former bound the responses to the dynamics of the business context, amplifying gains or fixing a lowest value to losses.(Dixit and Pindyck, 1994). The theoretical framework in which real options (RO) are analyzed arises with the Black-Scholes-Merton model (Black and Scholes, 1973), (Merton, 1973).

While financial problems are mostly analyzed through continuous-time models (Wilmott, 2009), the valuation of strategic flexibility has been carried out in discrete-time framework (Trigeorgis, 1997); (Amram and Kulatilaka, 1998); (Mun, 2004). , which are classical variants of the binomial model (Cox, Ross and Rubinstein, 1979). The uses of this model allow firms to increase gains and cut down losses (Smit and Trigeorgis, 2004).

To further compare the prescriptions of TCA and RO notice the both consider sequential decision-making under uncertainty (Williamson, 1985), (Trigeorgis and Mason, 1987), as well as the irreversibility and specific investment (Dixit and Pindyck, 1994), (Smith, 2005). But they differ in the underlying notions of rationality and their effects on how they handle uncertainty: while RO assumes full rationality and information processing capacity, TCA considers, as said, bounded rational agents. In the latter case, contracts are incomplete, since not all possible states are conceivable and consequently incorporated into contracts. But these differences allow complementarities between the two approaches. On one hand, TCA focuses on the protection against unexpected behaviors, reducing flexibility, while RO, on the other, provides coverage against environment uncertainty yielding more strategic alternatives. Our work will take the best from both approaches.

3. Real Options and Game-Theoretical Considerations: Provision Contracts

We will develop a model featuring all the aspects we intend to capture. Consider a input supply contract for which we will determine the benefits of the preservation of assets compared against the loss of flexibility. More precisely, we will contrast the current value of the contract with that of the option of changing to a potential alternative client. Since transactions costs are carried out in a discrete time we will use a binomial approach for the stochastic model of uncertainty. On the other hand, the agreement on payments and punishments for breakups are determined in as Nash equilibria in complete a perfect information games.

We break our analysis in three: **Case A** assumes a technologically stable environment, determining the value of the contract and the cost of breakup. Uncertainty of demand is obtained in a binomial model. **Case B** adds an option to changing to a new contract. The comparison of the values of the old contract and the option yields costs and benefits of renouncing to the former.

Finally, **Case C** introduces further flexibility into the contract, defining:

- (1) The minimal price to be agreed on with the new client, taking into account the costs involved in breaking up the original contract.
- (2) The optimal amounts to be supplied to both the old and new client, assuming that the prices and the plant capacity are fixed.

3.1 Case A: Agreement in a Stable Environment

Consider a supplier P providing some input to a client C who use is to make some final product. To provide this input, a previous investment I in the period t_0 is necessary, yielding benefits starting t_1 . This investment is highly specific and irreversible. It cannot be deferred and has no certain recovery value. The parties agree to carry out transactions for three periods, negotiating prices ex-ante. Suppose the agreed on unit price p of a unit of the final product in t_0 such that $p > c$, where the operation costs are c per unit. Thus, the benefits for P at any period t_1, t_2 and t_3 are $p - c$. The market value of the product v is such that $v > c$. The demand input is uncertain, but can modeled as a binomial process where the initial demand of C 's product is q . Two states are possible: a "good" one in which demand grows by a factor of $u > 1$, and a "bad" one in which the demand falls by a factor $d < 1$. The risk-less rate of interest is r per period. The risk-neutral probabilities are thus (Cox, J- Ross, S-Rubinstein, M, 1979);

$$p = \frac{(1+r)-d}{u-d} \quad (1)$$

and $1-p^2$.

² This mean that in time the demand evolve as follows:

t_0	t_1	t_2	t_3
			q^{u^3}
		q^{u^2}	q^{u^2d}
q	q^u	q^{ud}	q^{ud^2}
	q^d	q^{d^2}	q^{d^3}

and the expected demand at period t , $E(q)^t = \sum_{k=0}^t \frac{t!}{k!(t-k)!} p^k (1-p)^{t-k} q u^k d^{t-k}$

The benefits of a binding contract have to be compared to the results of only agreeing on an initial price without a long term commitment. Assume that both parties agree on an initial price \bar{p} per unit and that in the next three periods each party will try to capture the excess benefits, based on their respective bargaining powers. Furthermore, assume that while $v > \bar{p} > c$; $\bar{p} > (v + c)/2$; i.e \bar{p} is in the right half of the interval $[c, v]$. At t_1 , P worries that C , being his only customer, will try to get hold of the current value of P 's own benefits, offering a price $p=c$. On the other hand, C fears that the monopoly power of P will allow the latter to fixed a higher price $p=v$. Without external providers or customers, both are in a bilateral monopoly situation. In this case P and C might agree on keeping the pre-agreed price \bar{p} , or might agree in deviating, sharing the excess benefits in proportion to their bargaining power, which we assume is the same for both. Alternatively, one of the parties may try to impose its terms to the other. In the next periods the parties repeat the game, either agreeing to keep the original price or engaging in another round bargaining. The following matrix exhibits the strategies and the payoffs the players would get if the parties follow suit:

<i>P/C</i>	<i>Keep</i>	<i>Deviate</i>
<i>Keep</i>	$\bar{p} - c; v - \bar{p}$	$\bar{p} - v; v - c$
<i>Deviate</i>	$v - c; \bar{p} - v$	$(v - c)/2; (v - c)/2$

Table 1: Strategies and Payoffs for P and C (Own Elaboration)

Since $v - c > \bar{p} - c$, for P and $v - c > v - \bar{p}$; for both, the only Nash equilibrium (in dominant strategies) is that both players *Deviate*. The following matrix shows the corresponding asked prices at all four possible outcomes³:

<i>P/C</i>	<i>Keep</i>	<i>Deviate</i>
<i>Keep</i>	\bar{p}	—
<i>Deviate</i>	—	$v + c/2$

Table 2: Prices for all Possible Outcomes for P and C (Own Elaboration)

³ Notice that disagreements lead to break-ups of the contract, since the player that chooses to keep the original Price would not accept the terms of the other player.

To analyze the outcome in the repeated game consider the discounted cash flows of the agents, represented by the net present value of both agents, given C 's demand constraints:

$$NPV_P(sp, sc) = -I + \sum_{t=1}^3 \frac{1}{(1+r)^t} [E(q)^t B_p(sp, sc)] \quad (2)$$

$$NPV_C(sp, sc) = \sum_{t=1}^3 \frac{1}{(1+r)^t} [E(q)^t B_c(sp, sc)] \quad (3)$$

Where $sp, sc \in \{Keep, Deviate\}$ while $B_p(sp, sc)$ y $B_c(sp, sc)$ are the instantaneous unit benefits for P and C respectively, when they chooses sp and sc period t .

There are of course many cases that can be analyzed. But recall that unilateral deviation leads to the breakup of the contract and zero benefits for both parties. So we will focus on the cases in which either both agree in keeping \bar{p} , or both deviate, sharing in equal parts the excess benefits.

We have that $NPV_p(Deviate, Deviate) < NPV_p(Keep, Keep)^4$, and we assume that I is less than the discounted flow of benefits at least at $(Keep, Keep)$.⁵

But then, if C agrees on keeping the original price, P has incentives to deviate. On the other hand, NPV_c is larger in the stage Nash equilibrium than when both parties agree on keeping \bar{p} and thus has no incentive to agreeing on that.

The traditional way of addressing this in a repetition is by means of a *trigger strategy*, which punishes any move that goes against any desired result (Osborne and Rubinstein, 1994). Consider the case in which the original contract be enforced, i.e. $(Keep, Keep)$. We need to establish the appropriated punishments for both parties (M_p, M_c) , that make agreeing dominant strategy in the game and thus keep the price at \bar{p} . Consider the benefits at each period t when these penalties are enforced⁶:

⁴ Due to the condition on \bar{p} .

⁵ Since otherwise, it wouldn't be rational to sign the initial contract.

⁶ We assume that if only one party deviate, the penalty it pays is transferred to other.

<i>P/C</i>	<i>Keep</i>	<i>Deviate</i>
<i>Keep</i>	$\bar{p} - c; v - \bar{p}$	$M_c v - c - M_c$
<i>Deviate</i>	$v - c - M_p; M_p$	$(v - c/2) - M_p; (v - c/2) - M_c$

Table 3: Benefits and Penalties for P and C (own elaboration)

To ensure that (*Keep, Keep*) is the only Nash equilibrium, it suffices to fix M_p and M_c to be larger than $v - c$. That is, larger than the excess benefits of the transaction.

3.2 Case B: Agreement on the Provision in a Dynamic Environment

Let us assume now that the technological environment is dynamic, due to the entrance of new agents, in this case potential customers of *P*. As before, assume an agreement between *P* and *C*, at time t_0 . But at t_1 , *P* finds a potential new customer *Z*. Assuming that *P* has a limited capacity of provision, he has to decide on either respect the original agreement or to break it and make an agreement with *Z*. This can be seen as if *P* had an option that combines a long call and a put position. Breaking the agreement with *C* is like enabling a European sale option, while starting a new relation with *Z* is like activating a European buy option (*call*). The latter is exerted at t_2 , at which point *C* is dropped by *P*. The put has null exercise price while the new contract demands a marginal investment in the production facilities of *Z*, $I_z < I$ at t_1 ensuring the flow of resources to *Z* in t_2 and a further period t_3 . Given *Z*'s initial demand q_z its uncertainty is described also by a binomial process with rates u_z and d_z .

Whit an agreed on price \bar{p}_z , given the operational cost c_z , the instantaneous profit of *P* is $\bar{p}_z - c_z$. On the other hand, the value of one unit for *Z* is v_z . As before, we assume $v_z > \bar{p}_z > c_z$ whit $\bar{p}_z > (v_z + c_z)/2$.

The ensuing game between *P* and *Z* is summarized as follow (to be repeated in t_2 and t_3):

<i>P/Z</i>	<i>Keep</i>	<i>Deviate</i>
<i>Keep</i>	$\bar{p}_z - c_z; v_z - \bar{p}_z$	$\bar{p}_z - v_z; v_z - c_z$
<i>Deviate</i>	$v_z - c_z; \bar{p}_z - v_z$	$(v_z - c_z)/2; (v_z - c_z)/2$

Table 4: Strategies and Payoffs for P and Z in t_2 and t_3 (Own Elaboration)

If an agreement with Z is reached, the penalties for breaking up the contract with C (M_p, M_c), determined in a case **A**.

The combined option of P is exerted if the benefits plus the incremental investment and less the penalty for breaking up the agreement with C yield higher returns than the flows of funds expected from keeping the agreement with this client between t_2 and t_3 .⁷

$$\begin{aligned}
 & -(I + I_z) + \frac{1}{(1+r)} \sum_{k=0}^1 [p^k (1-p)^{t-k} q u^k d^{t-k} B_p(\text{Keep}^c, \text{Keep}_c)] + \\
 & \sum_{t=2}^3 \left[\sum_{k=0}^t \frac{t!}{k! (t-k)!} p^k (1-p)^{t-k} (q_z u_z^k d_z^{t-k} B_p(\text{Keep}^z, \text{Keep}_z) - M_p) \right] > \\
 & -I + \sum_{t=1}^3 \frac{1}{(1+r)^t} \left[\sum_{k=0}^t \frac{t!}{k! (t-k)!} p^k (1-p)^{t-k} q u^k d^{t-k} B_p(\text{Keep}^c, \text{Keep}_c) \right]
 \end{aligned}
 \tag{4}$$

This shows the *trade off* between respecting an original contract and using the strategic flexibility of options. While a contract reduces exposure to risk it also reduce the possibility of re contracting with a new client. AM_p defined in case **A** ensures that P will be able to enjoy the benefits of switching to Z , while P gets compensated for the period of break-up obtaining the equivalent to the highest possible benefit.

3.3 Case C: Further Flexibility

P can further try to size the largest possible share of the excess benefits in the negotiation with Z . This involving solving the follow problem:

$$\bar{p}_z \in \left[\frac{(v_z - c_z)}{2}, v_z \right] \max NPV_p(\text{breakup}), s. t NPV_z(\text{Keep}^z, \text{Keep}_z) > 0$$

Where

$$NPV_p(\text{breakup}) = -(I + I_z) + \frac{1}{(1+r)} \sum_{k=0}^1 [p^k (1-p)^{t-k} q u^k d^{t-k} B_p(\bar{p}_c - c)] +$$

⁷ Here $(\text{Keep}^c, \text{Keep}_c)$ represents the situation in which the original price \bar{p} is kept between P and C , while $(\text{Keep}^z, \text{Keep}_z)$ reflects the agreement between P and Z on \bar{p}_z

$$\sum_{t=2}^3 \frac{1}{(1+r)^t} \left[\sum_{k=0}^t \frac{t!}{k!(t-k)!} p^k (1-p)^{t-k} (q_z u_z^k d_z^{t-k} B_p(\bar{p}_z - c_z) - M_p \right] > (5)$$

And

$$NPV_z(Keep^z, Keep_z) = \sum_{t=2}^3 \frac{1}{(1+r)^{t-1}} \left[\sum_{k=0}^t \frac{t!}{k!(t-k)!} p^k (1-p)^{t-k} (q_z u_z^k d_z^{t-k} (v_z - \bar{p}_z)) \right] (6)$$

The linearity of the problem allows to reduce it to find \bar{p}_z such that $NPV_z(Keep^z, Keep_z) \approx 0$.⁸ That is $\bar{p}_z \approx v_z$.

Another possibility is for P instead of selling the product to only one client, to sell a fraction to each of them. That is, at t_2 provide a proportion f_c of the production to C and f_z to Z (i.e $f_c + f_z = 1$). Of course C and P will face a potential excess demand of their production, which in turn may impact on larger values on v and v_z . respectively. The contract only specifies the provision of amounts $f_c E(q)_c^t$ and $(1 - f_c) E(q)_z^t$ at prices \bar{p}_c and \bar{p}_z (where $E(q)_c^t$ and $E(q)_z^t$ are the expected demand of the final products of C and Z respectively).

It can be easy seen that the incentives to keeping or deviating from the contract with C and Z are the same as before. Thus the goal of P would be now:

$$\begin{aligned} & \max_{f_c \in (0,1)} NPV_p(f_c) \\ & \text{s. t } NPV_c(f_c, Keep_c) > 0 \text{ and } NPV_z(1 - f_c, Keep_z) > 0 \end{aligned}$$

Where

$$NPV_p(f_c) = -(I + I_z) + \frac{1}{(1+r)} \sum_{k=0}^1 [p^k (1-p)^{t-k} q u^k d^{t-k} (\bar{p}_c - c)] +$$

⁸ We intend an expression $x \approx y$ to mean that $x \neq y$ but $|x - y|$ close to 0.

$$\sum_{t=2}^3 \frac{1}{(1+r)^t} \left[\sum_{k=0}^t \frac{t!}{k!(t-k)!} p^k (1-p)^{t-k} (qu^k d^{t-k} (\bar{p}_c - c)) + (q_z u_z^k d_z^{t-k} (1-f_c) (\bar{p}_z - c_z)) \right] \quad (7)$$

While

$$NPV_c(f_c, Keep_z) = \frac{1}{(1+r)} \sum_{k=0}^1 [p^k (1-p)^{t-k} qu^k d^{t-k} (v_c - \bar{p}_c)] + \sum_{t=2}^3 \frac{1}{(1+r)^t} \left[\sum_{k=0}^t \frac{t!}{k!(t-k)!} p^k (1-p)^{t-k} (q_z u_z^k d_z^{t-k} f_c (v_c - \bar{p}_c)) \right] \quad (7)$$

And

$$NPV_z(Keep^z, Keep_z) = \sum_{t=2}^3 \frac{1}{(1+r)^t} \left[\sum_{k=0}^t \frac{t!}{k!(t-k)!} p^k (1-p)^{t-k} (q_z u_z^k d_z^{t-k} (1-f_c) (v_z - \bar{p}_z)) \right] \quad (8)$$

Again, the linearity of the problem reduces it to the comparison between $\bar{p}_c - c$ and $\bar{p}_z - c_z$. That is the optimal level f_c^* is $\simeq 1$ if $(\bar{p}_c - c) > (\bar{p}_z - c_z)$; $\simeq 0$ if $(\bar{p}_c - c) < (\bar{p}_z - c_z)$ and $\frac{1}{2}$ otherwise.

4. Conclusions

We have the pros and cons of using RO approach to contracts. We compare it to the rigidity predicated by the Neo-Institutional line of thought that sees flexibility as a source of additional transaction costs. We illustrated this comparison in the light of a model-client problem. We saw that adequate penalties enforce relation if not outsider parts exist, but allow break-up of the relation to seek out opportunities. This possibility of switching partners can be fully captured in a real options framework and the optimal values can be assessed through game theoretical-analyses.

These formal explorations have been carried out assuming the full rationality of the involved parties and common knowledge of all relevant futures events. We think that the advantages of the RO approach still stand if we drop these assumptions and change towards a *behavioral* set of hypothesis (Kahneman and Tversky, 1979);(Shefrin, 2010), in which the agents use heuristics instead of seeking optimal solutions. Further work involves exploring this intuition.

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